### Problem I–1

Determine all real numbers $A$ such that every sequence of non-zero real numbers $x_1, x_2, \ldots$ satisfying

$$x_{n+1} = A - \frac{1}{x_n}$$

for every integer $n \geqslant 1$, has only finitely many negative terms.

### Problem I–2

Let $m$ and $n$ be positive integers. Some squares of an $m \times n$ board are coloured red. A sequence $a_1, a_2, \ldots, a_{2r}$ of $2r \geqslant 4$ pairwise distinct red squares is called a *bishop circuit* if for every $k \in \{1, \ldots, 2r\}$, the squares $a_k$ and $a_{k+1}$ lie on a diagonal, but the squares $a_k$ and $a_{k+2}$ do not lie on a diagonal (here $a_{2r+1} = a_1$ and $a_{2r+2} = a_2$).

In terms of $m$ and $n$, determine the maximum possible number of red squares on an $m \times n$ board without a bishop circuit.

(*Remark.* Two squares lie on a diagonal if the line passing through their centres intersects the sides of the board at an angle of $45°$.)

### Problem I–3

Let $ABC$ be an acute triangle and $D$ an interior point of segment $BC$. Points $E$ and $F$ lie in the half-plane determined by the line $BC$ containing $A$ such that $DE$ is perpendicular to $BE$ and $DE$ is tangent to the circumcircle of $ACD$, while $DF$ is perpendicular to $CF$ and $DF$ is tangent to the circumcircle of $ABD$. Prove that the points $A$, $D$, $E$ and $F$ are concyclic.

### Problem I–4

Let $n \geqslant 3$ be an integer. Zagi the squirrel sits at a vertex of a regular $n$-gon. Zagi plans to make a journey of $n-1$ jumps such that in the $i$-th jump, it jumps by $i$ edges clockwise, for $i \in \{1, ..., n-1\}$. Prove that if after $\lceil \frac{n}{2} \rceil$ jumps Zagi has visited $\lceil \frac{n}{2} \rceil + 1$ distinct vertices, then after $n-1$ jumps Zagi will have visited all of the vertices.

(*Remark.* For a real number $x$, we denote by $\lceil x \rceil$ the smallest integer larger or equal to $x$.)

Time: 5 hours
Time for questions: 60 min
Each problem is worth 8 points.
The order of the problems does not depend on their difficulty.